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## Numerical evaluation of field profile in an undulator with bulk HTS

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Application of a bulk high-T<sub>c</sub> superconductor (HTS) to a permanent magnet undulator is numerically evaluated using macroscopic numerical simulation based on the critical state model. Shielding currents are induced by field-cooled magnetization with increasing of the gap length between magnets of the undulator. A hole of the HTS ring is treated as low conductivity region. Magnetic field at the center of the undulator is compared for cases with and without the bulk HTS. Numerical results agree well with the experimental results. Shielding field of the HTS ring is also numerically evaluated and discussed with experimental results.

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**Keywords** : high-T<sub>c</sub> superconductor; permanent magnet undulator; macroscopic numerical simulation; critical state model

**1. Introduction**

Permanent magnet (PM) undulators are important devices in synchrotron radiation (SR) facilities [1]-[2]. Fig. 1 shows a schematic model of the PM undulator. Magnetic circuits are formed by magnet modules with PM and magnetic adjustments. Periodically reserved magnetic fields are obtained in a gap between the PM modules. When an electron is vertically injected to the field, it rotates by synchrotron motion. The high speed electron wiggles like a broken arrow in the periodically reserved fields of the PM undulator, and it radiates a part of its energy as SR. Wave length of the SR is shown as follows with period  $T$  of the undulator and energy  $\gamma$  of the electron beam [3],

$$\lambda = \frac{T}{2\gamma^2} \left(1 + \frac{K^2}{2}\right), \quad K = \frac{eB_0T}{2\pi mc}, \quad (1)$$

where  $e$ ,  $m$ ,  $c$  are charge, mass, velocity of electron and  $B_0$  is magnetic field of the undulator. Since the undulator parameter  $K$  is about 1.0 in the SR facilities, higher energy  $\gamma$  or shorter period  $T$  are needed to obtain short wave length of the X-ray SR. Radius of the synchrotron motion depends on magnitude of magnetic field. Application of a bulk high-T<sub>c</sub> superconductor (HTS) to the undulator was reported in the SPring-8, the largest SR facility in Japan, to obtain a short period of SR by high magnetic field by the HTS [1]-[2]. Shielding currents are induced in the HTS by field-cooled magnetization with increasing of the gap length between the magnets of the undulator.

In the present study, the field profiles in the PM undulator with the bulk HTS are numerically evaluated in the analysis of the field-cooled magnetization. Shielding currents in the bulk HTS are analyzed by macroscopic numerical simulation using the critical state model with thin plate-multi layers approximation. Field profiles in the center of the undulator are compared for cases with and without the bulk HTS. Numerical results agree well with the reported

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experimental results. Shielding field of the HTS ring is also numerically evaluated and is discussed with experimental results.

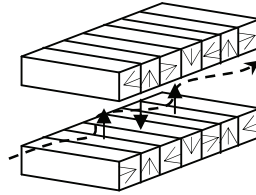


Fig. 1. Simple model of a PM undulator

## 2. Numerical formulation

Macroscopic electromagnetic phenomena in HTS are described by Maxwell equations [4]-[6] :

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0, \quad (2)$$

where  $\mu_0$ ,  $\mathbf{E}$  and  $\mathbf{B}$  are the magnetic permeability in air, and the electric and magnetic fields, respectively. The magnetic field  $\mathbf{B}$  is caused by the external current  $\mathbf{J}_0$  and the shielding current  $\mathbf{J}_{SC}$ . The shielding currents are obtained based on the standard critical state model, where constitutive relationships between the shielding current density  $\mathbf{J}_{SC}$  and the electric field  $\mathbf{E}$  are obtained from the force balance on a fluxoid :

$$\mathbf{J}_{SC} = J_c \left( \frac{\mathbf{E}}{|\mathbf{E}|} \right) \quad (\text{if } |\mathbf{E}| \neq 0), \quad \frac{\partial \mathbf{J}_{SC}}{\partial t} = \mathbf{0} \quad (\text{if } |\mathbf{E}| = 0). \quad (3)$$

When the electric field  $\mathbf{E}$  is induced in a local region by change of the magnetic field, shielding currents with the critical current density  $J_c$  are obtained. If there is no electric field by the shielding effect, the situation of currents is not changed. Though the critical current density  $J_c$  has a strong dependence on magnetic field, Bean model is applied to the present analysis since the critical current density is almost constant in the low cryogenic temperature in experiments. Shielding current distributions are evaluated using current vector potential  $\mathbf{T}$ , defined by  $\mathbf{J}_{SC} = \nabla \times \mathbf{T}$ . Under a thin plate approximation, governing equation in the present analysis is obtained as follows [5]-[6] :

$$\frac{1}{\sigma} \nabla^2 T - \mu_0 \frac{\partial T}{\partial t} - \frac{\mu_0}{4\pi} \int_S \frac{\partial T_n}{\partial t} \nabla' \frac{1}{R} dS' = \frac{\partial B_0}{\partial t}, \quad (4)$$

where  $T_n$  is a normal component of  $\mathbf{T}$  on the surface. The shielding current distribution in the superconductor is obtained stably using the following numerical technique with artificial conductivity [5]-[6]. At first, conductivity in all elements is set to very large value, e.g.  $10^{20} [1/\Omega\text{m}]$ , assuming the superconductor is a very good conductor. If current over the critical current density  $J_c$  is obtained, the conductivity of the element is corrected as follows:

$$\sigma_{\text{new}} = \frac{J_c}{E} = \sigma_{\text{old}} \frac{J_c}{J} \quad \text{if } J > J_c, \quad \sigma_{\text{new}} = \sigma_{\text{old}} \quad \text{if } J \leq J_c. \quad (5)$$

Numerical model is discretized using finite element method. Matrix equation is resolved in iterative calculations in each time step, until the maximum current is converged to the critical current density. The obtained self-consistent shielding current distribution is almost satisfied with the non-linear constitutive relationships in (3).

## 3. Numerical results

A part of the lower PM module is shown in Fig. 2, where size of the PM and the magnetic adjustments are 20x4x20mm and 20x3.5x17mm, respectively. Size of the HTS ring is 32x7x3mm with a hole of 20x4mm. The maximum magnetic field without HTS ring is about 1.0 T with 5.0mm gap length. The permanent magnets are expressed using surface currents in the numerical analysis.

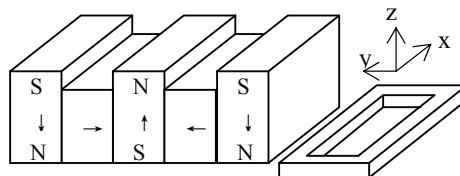


Fig. 2. Permanent magnet module and a HTS ring with a hole

The HTS ring is set to the top of the permanent magnet and field-cooled. The gap length between lower and upper PM modules is increased from zero to 5.0 mm as shown in Fig.3. Shielding currents are induced change of the magnetic field for the HTS ring. A model of the HTS ring with a hall is divided 30x30x2 triangle elements with 7 layers. The hole is treated as low conductivity region. Fig. 4 shows comparison of experimental [1]-[2] and numerical magnetic fields on the center with 5.0mm gap length, where Figs. (a) and (b) are the cases with  $J_c=1.0 \times 10^8$  A/m<sup>2</sup> and  $J_c=5.0 \times 10^8$  A/m<sup>2</sup>, respectively. Numerical result of the case with  $J_c=5.0 \times 10^8$  agrees well with the experimental result. 1.15 times larger peak field is obtained in the case with the HTS ring.



Fig. 3. Field-cooled magnetization by increasing of the gap length

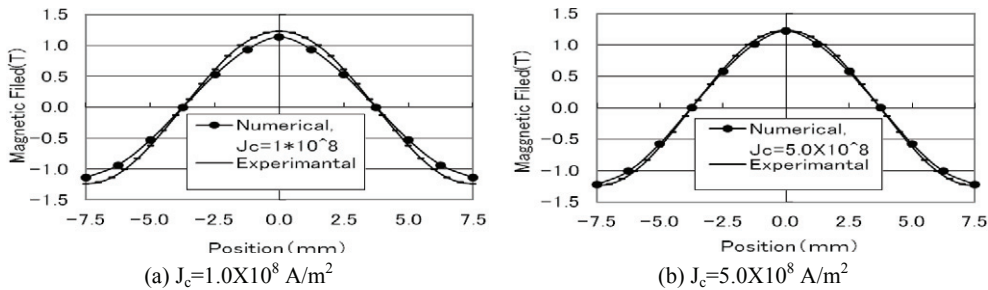


Fig. 4. Magnetic field profile with HTS for 5.0mm gap length

Fig. 5 shows comparison of experimental and numerical magnetic fields at the center point of the gap, where gap length is changed from 3.0 mm to 20.0 mm. Figs. (a) and (b) are experimental and numerical results, and dash and solid lines show the cases with and without the HTS, respectively. Decreasing tendency of the field against the gap length is almost the same for both cases. Almost the same dependence is obtained for the case with  $J_c=5.0 \times 10^8$  A/m<sup>2</sup>.

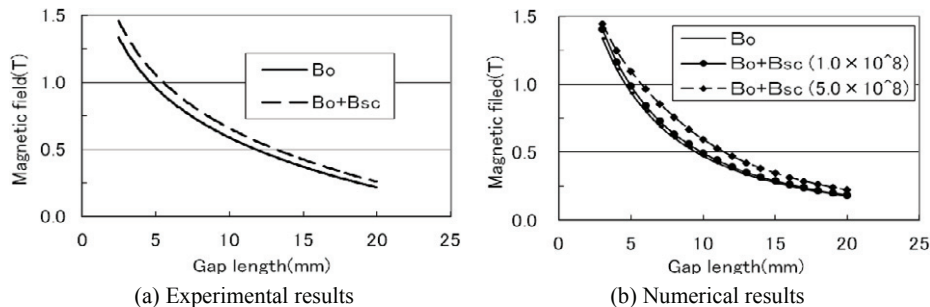


Fig. 5. Magnetic fields for different gap lengths: dash and solid lines show the cases with and without the HTS ring.

Another experimental result of zero field cooled magnetization was also reported by Tanaka et.al. [1]. A HTS ring was placed at the center of electromagnet, and magnetic field applied to the HTS ring from zero to 2.0 T. Shielding field induced by the shielding current of the HTS ring was evaluated from field measurements with Hall probe. Fig. 6 shows shielding field of center of the ring, where dash and solid lines show experimental [1] and numerical results, respectively. The same tendency is obtained for the case with  $J_c=1.0 \times 10^9$  A/m<sup>2</sup> in this experimental case. There is a peak in the shielding field during magnetization. Fig. 7 shows field profile of numerical results in Fig. 6. Since the HTS ring is long and narrow shape, there are two peaks of shielding field at both edge of the HTS ring. Maximum shielding field at center is obtained when applied field is 1.6 T, and it decreases a little at 2.0 T. Fig. 8 shows the HTS ring. Cross sections of point A and B are 1.75x3.0mm and 6.0x3.0mm, respectively. Maximum shielding current of the whole HTS ring is obtained when the shielding current flow all cross section of the point A. After that, local shielding current induced the point B, and fields of two peaks are increased and field of center is decreased.

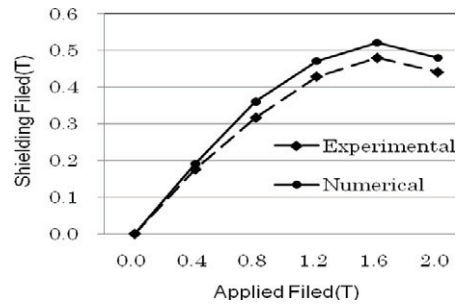


Fig. 6. Shielding field at center of HTS. Dash and solid lines show experimental and numerical results.

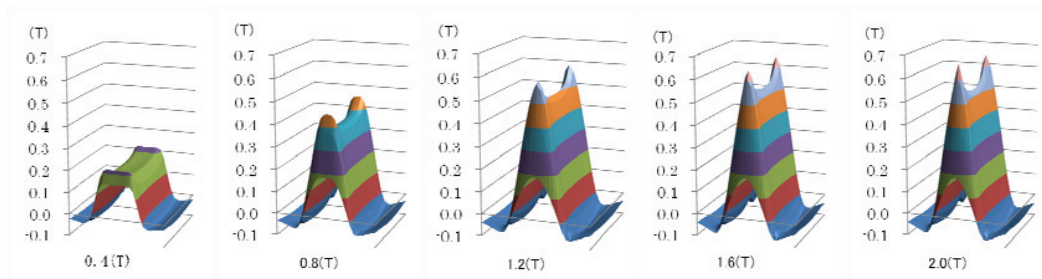


Fig. 7. Field profile of numerical results in Fig. 6.

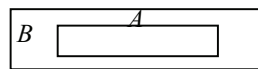


Fig. 8. HTS ring of 32x7x3mm with a hole of 20x4mm.

#### 4. Conclusion

The field profiles in the PM undulator with the bulk HTS are numerically evaluated in the analysis of the field-cooled magnetization. Shielding currents in the bulk HTS are analyzed by macroscopic numerical simulation using the critical state model with thin plate-multi layers approximation. Field profiles in the center of the undulator are compared for cases with and without the bulk HTS. Numerical results agree well with the reported experimental results. Shielding field of the HTS ring with a hole is also numerically evaluated and discussed with experimental results. Numerical evaluation of field profile of another bulk HTS undulator, where semicircle bulk HTSs are arranged by turns in solenoid coil [3], will be carried out in near future.

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